PB82-154733

Phosphorus Retention Capacity of Lakes.

(U.S.) Corvallis Environmental Research Lab., OR

1976

U.S. Department of Commerce
National Technical Information Service
NTIS.

TECHNICAL REPORT DATA
(Please read Instructions on the reverse before completing)

1. REPORT NO.	2.	3.
EPA-600/J-76-112	Journal Article	PB82-154733

4. TITLE AND SUBTITLE
Phosphorus Retention Capacity of Lakes

5. REPORT DATE
January 1982

6. PERFORMING ORGANIZATION CODE
029064

7. AUTHOR(S)
D.P. Larsen and H.T. Mercier

8. PERFORMING ORGANIZATION REPORT NO.

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Environmental Research Laboratory
Office of Research and Development
U.S. Environmental Protection Agency
Corvallis, Oregon 97333

10. PROGRAM ELEMENT NO.

11. CONTRACT/GRANT NO.

12. SPONSORING AGENCY NAME AND ADDRESS
same

13. TYPE OF REPORT AND PERIOD COVERED
inhouse

14. SPONSORING AGENCY CODE
EPA/600/02

15. SUPPLEMENTARY NOTES
published in Journal of the Fisheries Research Board of Canada 33(8):1742-1750, 1976

16. ABSTRACT

Mean annual lake phosphorus concentrations ($[P]_x$) in a steady state system can be described as a function of two variables, the mean annual influent P concentration and the net annual P sedimentation: $[P]_x = [\bar{p}] (1 - R_v)$. $[\bar{p}]$ is the annual P supply/annual water supply and R_v is that fraction of inflowing P retained by the sediments annually. Since R_v is critical in determining $[P]_x$ lake properties were examined for their influence on R_v; the best fit empirical expressions developed related R_v to either areal water supply (annual water inflow rate/lake surface area) or hydraulic washout coefficient (annual water outflow rate/lake volume). The expressions were derived from characteristics of low productivity lakes; their validity for more productive lakes was not examined. A graph of $[\bar{p}]$ vs R_v is proposed to illustrate the relationship between $[P]_x$, $[\bar{p}]$, and R_v and if lake P concentrations are used as measures of trophic state, this graph is useful for predicting trophic state and changes in trophic state caused by altering $[\bar{p}]$ and/or R_v. It is conceptually similar to those of Vollenwider (1975) and Dillon (1975) but differs in that it separates the potential lake concentration ($[P]_x$ would be equal to $[\bar{p}]$ if P were conservative) from those factors acting to reduce $[\bar{p}]$, expressed as R_v to produce lake concentrations.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group

18. DISTRIBUTION STATEMENT
Release to public

REPRODUCED BY
NATIONAL TECHNICAL INFORMATION SERVICE
U.S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA 22161

19. SECURITY CLASS (This Report)	21. NO. OF PAGES
unclassified	
20. SECURITY CLASS (This page)	**22. PRICE**
unclassified	

Reprinted from

Journal of the Fisheries Research Board of Canada

Réimpression du

Journal de l'office des recherches sur les pêcheries du Canada

095J

EPA-600/J-76-112
Journal Article

Phosphorus Retention Capacity of Lakes

D. P. Larsen and H. T. Mercier

Volume 33 • Number 8 • 1976

Pages 1742–1750

Environment Canada
Fisheries and Marine Service

Environnement Canada
Service des pêches et des sciences de la mer

Phosphorus Retention Capacity of Lakes

D. P. LARSEN AND H. T. MERCIER

U.S. Environmental Protection Agency, Ecosystems Modeling and Analysis Branch, Corvallis Environmental Research Laboratory, Corvallis, Oreg. 97330, USA

LARSEN, D. P., and H. T. MERCIER. 1976. Phosphorus retention capacity of lakes. J. Fish. Res. Board Can. 33: 1742–1750.

Mean annual lake phosphorus concentrations ($[P]_x$) in a steady state system can be described as a function of two variables, the mean annual influent P concentration and the net annual P sedimentation: $[P]_x = [\bar{p}] (1 - R_p)$ (see Rigler 1975). $[\bar{p}]$ is the annual P supply/annual water supply and R_p is that fraction of inflowing P retained by the sediments annually. Since R_p is critical in determining $[P]_x$, lake properties were examined for their influence on R_p; the best fit empirical expressions developed related R_p to either areal water supply (annual water inflow rate/lake surface area) or hydraulic washout coefficient (annual water outflow rate/lake volume). The expressions were derived from characteristics of low productivity lakes; their validity for more productive lakes was not examined.

A graph of $[\bar{p}]$ vs. R_p is proposed to illustrate the relationship between $[P]_x$, $[\bar{p}]$, and R_p, and, if lake P concentrations are used as measures of trophic state, this graph is useful for predicting trophic state and changes in trophic state caused by altering $[\bar{p}]$ and/or R_p. It is conceptually similar to those of Vollenweider (1975) and Dillon (1975) but differs in that it separates the potential lake concentration ($[P]_x$ would be equal to $[\bar{p}]$ if P were conservative) from those factors acting to reduce $[\bar{p}]$, expressed as R_p, to produce lake concentrations.

LARSEN, D. P., and H. T. MERCIER. 1976. Phosphorus retention capacity of lakes. J. Fish. Res. Board Can. 33: 1742–1750.

Les concentrations moyennes annuelles de phosphore dans un lac ($[P]_x$) dans un système en état d'équilibre peuvent être décrites comme étant fonction de deux variables, la concentration moyenne annuelle du P d'entrée et la sédimentation annuelle nette du P : $[P]_x = [\bar{p}] (1 - R_p)$ (voir Rigler, 1975). $[\bar{p}]$ est le quotient apport annuel de P/apport annuel d'eau et R_p est cette fraction du P d'entrée qui est retenue par le sédiment en un an. Etant donné que R_p joue un rôle critique dans la détermination de $[P]_x$, nous avons étudié les propriétés du lac en vue de déterminer leur influence sur R_p; d'après les meilleures exressions développées empiriquement, R_p est fonction soit des apports d'eau par superficie (taux annuel d'entrée d'eau/superficie de la surface du lac) soit du coefficient de lavage hydraulique (taux annuel de sortie d'eau/volume du lac). Nous avons établi ces formules à partir des caractéristiques de lacs peu productifs; nous n'avons pas vérifié leur validité pour des lacs plus productifs.

Nous proposons un graphique de $[\bar{p}]$ vs. R_p pour illustrer la relation entre $[P]_x$, $[\bar{p}]$, et R_p. Si les concentrations de P dans le lac sont utilisées comme mesures de l'état trophique, ce graphique permet également de prédire l'état trophique et les changements qu'entraîne dans l'état trophique une altération de $[\bar{p}]$ et/ou de R_p. Dans son concept, le graphique est semblable à ceux de Vollenweider (1975) et de Dillon (1975). Il en diffère cependant en ceci qu'il sépare la concentration potentielle dans le lac ($[P]_x$ serait égale à $[\bar{p}]$ si P était faible) des autres facteurs causant une diminution de $[\bar{p}]$, exprimé sous forme de R_p, pour produire les concentrations dans le lac.

Received August 12, 1975
Accepted May 11, 1976

Reçu le 12 août 1975
Accepté le 11 mai 1976

MASS balance models have been developed to describe the relationship between phosphorus (P) concentrations in lakes and the supply of P to lakes [illegible]ow by Dillon 1974; Imboden 1973, 1974; Lerman 1974; Lorenzen 1973; Snodgrass and O'Melia 1975; Sonzogni et al. 1974; Vollenweider 1969, 1975). Because lake P concentrations can be used as an indication of trophic state (Sawyer 1947; Vollenweider 1968; Dillon 1975), these models can be useful for predicting water quality and changes in water quality associated

Printed in Ca[illegible] (J3948)
Imprimé au Canada (J3948)

with changes in P supply. In the simplest form, these models state that on an annual basis the change in P content of lakes is related to P inflow, outflow, and sedimentation:

$$\frac{dP}{dt} = J - Q\cdot[P] - S \qquad (1)$$

where [P] is the mean P concentration in the lake; J, the rate of external P supply; Q, the water outflow rate; and S, the net P sedimentation rate. The steady state solution to equation (1) is:

$$[P]_{\infty} = \frac{J - S}{Q} \qquad (2)$$

Assumptions underlying equation (1) are: rate of water inflow and outflow are steady and equal; the system is well mixed at all times; and a net loss of P to the sediments occurs on an annual basis.

The net fraction of inflowing P deposited on the sediments is S/J and has been defined by Vollenweider (1975) in a slightly different way as the P retention coefficient, R_p. Thus, substituting $J\cdot R_p$ for S, equation (2) can be written:

$$[P]_x = [\bar{p}]\ (1 - R_p) \qquad (3)$$

where $[\bar{p}] = J/Q$, the average influent P concentration (see Riger 1975). These equations describe the continuity of mass for P independent of the mechanisms by which the sedimentation term is described.

Equation (3) demonstrates the dependence of mean lake P concentrations on two variables, the mean influent P concentration, and the P assimilative capacity of lakes. We constructed a graph of $[\bar{p}]$ vs. R_p to show this relationship using data available from Dillon and Rigler (1974), Dillon (1975), and J. Gakstatter (Corvallis Environmental Research Laboratory, Corvallis, Oreg., 1974, personal communication): Although lakes generally violate some or all of the assumptions underlying the basic model, their trophic states agree well with those predicted from this relationship. This graph is similar in concept to those of Vollenweider (1975) and Dillon (1975). It is useful for predicting trophic state and changes in trophic state caused by manipulating influent P concentrations and/or manipulating a lake's P assimilative capacity.

The importance of the P retention capacity as a determinant of steady state lake concentrations is evident from equation (3). An examination of factors that control its magnitude is thus valuable for theoretical as well as practical purposes. Kirchner and Dillon (1975) and Chapra (1975) showed that the retention capacity could be related to areal water loading (q_s, annual water inflow rate/lake surface area). We present several other expressions that provide good statistical correlations between observed and predicted values of P retention capacity in lakes of low productivity. These relate the P retention capacity to either areal water loading or to the hydraulic washout coefficient ($\rho_w = Q/V$; V = lake volume).

Data Base

Three sources were used to obtain data to plot the position of lakes on a $[\bar{p}]$ vs. R_p graph and to determine relationships between R_p and lake properties. Dillon and Rigler (1974) and Dillon (1975) included estimates of L_p (areal P loading, g P/m^2 per yr), ρ_w (yr^{-1}), $\bar{z}$ (mean depth, meters) and R_{exp} for various lakes in North America and Europe. The quotient, $L_p/(\rho_w\cdot\bar{z})$ is assumed equivalent to $[\bar{p}]$ and $\rho_w\cdot\bar{z}$ equivalent to q_s because differences between water outflow and inflow rates are probably small. Vollenweider (1975) defined R_p as $\frac{\sigma_p}{\sigma_p + \rho_w}$ where σ_p (yr^{-1}) was the P deposition coefficient he hypothesized to describe P sedimentation as a function of the amount of P present in the lake as: $d[P]/dt = J/V - (\rho_w + \sigma_p)[P]$. Because a direct determination of its value is difficult to obtain and has not been determined for many lakes, R_p has generally been calculated as: $R_{exp} = 1 - \frac{\text{outflow load}}{\text{inflow load}}$ (Dillon and Rigler 1974). The distinction between R_p (as a theoretical value or as estimated by sedimentation measurements) and R_{exp} (as experimentally estimated from P inflow and outflow) will be maintained here, although they are equal at steady state if model assumptions are met. J. Gakstatter (Corvallis Environmental Research Laboratory, Corvallis, Oreg., 1974, personal communication) provided another source of data that includes ρ_w and $\bar{z}$, as well as inflow and outflow P concentrations and loadings, for many lakes in northeastern and midwestern United States studied by the U.S. Environmental Protection Agency's National Eutrophication Survey. Tables 1 and 2 summarize the data used.

The trophic status of lakes listed in Tables 1 and 2 were classified as O (oligotrophic), M (mesotrophic), E (eutrophic) and HE (hypereutrophic). These classifications are qualitative, based upon the assessments of limnologists who have investigated these lakes. In some cases, the identity of trophic state was not available.

$[\bar{p}]$ vs. R_{exp} Graph

Figure 1 was constructed from the data in

TABLE 1. Summary of lakes and characteristics used in Fig. 1. Data for lakes 1–17 were obtained from Dillon (1975) and references therein and for lakes 18–37 from J. Gakstatter (personal communication 1974).

No.	Lake	Trophic state	$[\bar{p}]$ ($\mu g/liter$)	R_{exp}	No.	Lake	Trophic state	$[\bar{p}]$ ($\mu g/liter$)	R_{exp}
1	Baldeggersee	E	230	0.61	19	Bemidji, Minn.	E	35	0.36
2	Bodensee-Obersee	E	200	0.65	20	Blackduck, Minn.	E	135	0.55
3	Erie	M-E	150	0.84	21	Chemung, Mich.	E	105	0.63
4	Greifensee	E	170	0.62	22	Clearwater, Minn.	E	180	0.76
5	Hallwilersee	O	75	0.36	23	Cokato, Minn.	E	385	0.44
6	Kalamalka	E	610	0.90	24	Delavan, Wis.	E	410	0.55
7	Kegonsa	E	505	0.09	25	Fremont, Wis.	HE	550	0.47
8	Monona	E	330	0.71	26	Green, Minn.	M-E	55	0.67
9	Michigan	O-M	85	0.9	27	Jordan, Mich.	E	130	0.17
10	Norrviken (1961–62)	E	425	0.49	28	Le Homme Dieu, Minn.	E	135	0.78
11	Okanagan	O	305	0.95	29	Nagawicka, Wis.	E	205	0.57
12	Pfaffikersee	E	195	0.77	30	Nest, Minn.	E	90	0.56
13	Sebasticook	E	90	0.48	31	Sebasticook, Maine	E	60	0.29
14	Skaha	M-E	95	0.65	32	Shawano, Wis.	E	35	0.23
15	Waubesa	E	620	0	33	Swan, Wis.	E	140	0.18
16	Wood	E	2590	0.90	34	Townline, Wis.	E	255	0.52
17	227 (ELA)	E	325	0.89	35	Trout, Minn.	E	375	0.92
					36	Wagonga, Minn.	E	4300	0.57
18	Beaverdam (Dodge County, Wis.)	E	265	0.0	37	Wapogasset, Wis.	E	70	0.43

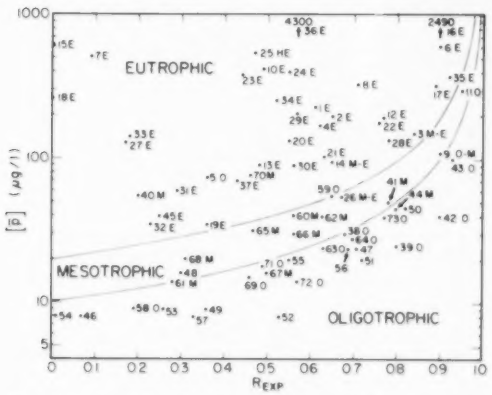

FIG. 1. Graph of mean tributary phosphorus concentration ($[\bar{p}]$) vs. phosphorus retention coefficient (R_{exp}) for lakes summarized in Tables 1 and 2. O = oligotrophic; M = mesotrophic; E = eutrophic; HE = hypereutrophic. Lines are solutions of equation (3) for $[\bar{p}]$ and R_{exp} values that produce lake concentrations of 10 and 20 μg/liter.

Tables 1 and 2, showing the modifying effect of a lake's P retention capacity upon the influent P concentration. Lines of constant lake P concentration can be drawn as solutions of Equation (3) for particular values of $[p]$ and R_p. If particular lake mean P concentrations can be used to designate the trophic state of a lake, curves delineating trophic states can be drawn, as adapted from Lerman (1974) and Dillon (1975). Dillon used lake concentrations of 10 and 20 μg/liter to separate oligotrophic (<10 μg/liter), mesotrophic (10–20 μg/liter) and eutrophic (>20 μg/liter) conditions. The same designations are used here.

The positions of the lakes on Fig. 1 agree reasonably with their trophic states. Some lakes violate the pattern. Several reasons can be suggested for their departure from the model and a more detailed investigation would be necessary to determine the cause of this discrepancy. Reasons for the discrepancies might be:

1) Observed lakes might not be in or near a steady state and, consequently, recent changes in $[\bar{p}]$ may not yet be reflected by changes in lake characteristics.

2) Data base for the calculations might be inadequate for the present analysis which requires an adequate estimate of the mean concentration of P in inflowing and outflowing waters. The optimum method for obtaining P flux into or out of lakes has only recently been critically evaluated (Treunert et al. 1974). Thus, the data reflect the notions of the principal investigators so sampling frequency may be neither consistent nor optimal.

3) Characteristics of some lakes may sufficiently disagree with model assumptions to invalidate a comparison. For example, vertical and horizontal inhomogeneities might be large enough

TABLE 2. Summary of lakes and characteristics used in Fig. 1 and for an analysis of relationships between R_{exp} and lake characteristics. Data for lakes 38–45 were obtained from Dillon (1975) and references therein, for lakes 46–57 from Dillon and Rigler (1974), and for lakes 58–73 from J. Gakstatter (personal communication 1974). Asterisks indicate lakes used for generating best empirical relationships between R_{exp} and lake characteristics.

No.	Lake	Trophic state	$[\bar{p}]$ ($\mu g/liter$)	R_{exp}	ρ_w (yr^{-1})	$\bar{z}$ (m)	σ_p (yr^{-1})
38	Aegerisee	O	30	0.68	0.115	49	0.24
39	*Clear	O	25	0.80	0.13	12.5	0.52
40	Leman	M	55	0.20	0.083	155	0.02
41	Ontario	M	50	0.78	0.152	84	0.54
42	*Superior	O	40	0.90	0.0053	148	0.05
43	*Tahoe	O	100	0.93	0.0014	303	0.02
44	Turlersee	M	45	0.80	0.465	14	1.86
45	Zurichsee	E	40	0.25	0.680	50	0.23
46	*Beech		8	0.07	22.7	9.8	1.71
47	*Bob		24	0.71	0.37	18.0	0.91
48	*Cameron		16	0.30	18.9	7.1	8.1
49	*Eagle-Moose		9	0.36	2.03	12.8	1.14
50	Four Mile		45	0.82	0.26	9.3	1.18
51	*Oblong-Haliburton		20	0.72	0.32	17.7	0.82
52	*Halls		8	0.53	0.96	27.2	1.08
53	*Maple		9	0.26	8.0	11.6	2.81
54	*Pine		8	0.01	18.5	7.4	0.19
55	Raven		20	0.55	14.9	0.73	18.2
56	Talbot		24	0.69	4.9	0.85	10.9
57	*Twelve Mile — Boshkung		8	0.33	2.38	18.1	1.18
58	*Bay of Naples, Maine	O	9	0.19	14.0	4.3	3.28
59	Canadaigua, New York	O	55	0.65	0.07	39.0	0.13
60	Carlos, Minn.	M	40	0.56	0.27	13.1	0.34
61	*Carry Falls, New York	O	14	0.28	9.61	5.4	3.74
62	Cass, Minn.	M	40	0.63	1.17	7.6	1.99
63	*Charlevoix, Mich.	O	24	0.63	0.31	16.8	0.53
64	Higgins, Mich.	O	28	0.70	.06	14.9	0.14
65	Houghton, Mich.	M	32	0.47	0.76	2.3	0.68
66	Long (Aroostook County, Maine)	M	30	0.56	0.31	13.4	0.39
67	*Long (Cumberland County, Maine)	M	16	0.50	0.83	10.4	0.83
68	*Mattawamkegg, Maine	M	20	0.31	8.69	3.7	3.90
69	*Moosehead, Maine	O	15	0.46	0.33	16.4	0.28
70	Pelican, Minn.	M	77	0.46	0.30	2.7	0.25
71	*Rangeley, Maine	O	18	0.49	0.36	14.3	0.35
72	*Sebago, Maine	O	14	0.57	0.19	30.8	0.25
73	Winnipesaukee, Minn.	O	40	0.77	0.25	13.1	0.84

to seriously affect the model assumption of a completely and instantly mixed system, or factors other than phosphorus might control trophic state.

4) Identity of trophic state is subjective so that designations are not likely to be universally acceptable.

Phosphorus Retention Capacity

Determining the P retention capacity is critical to the successful prediction of lake P concentrations. Empirically, R_p can be estimated from the ratio of outflow loads or concentrations to inflow loads or concentrations. Therefore, experimental determination of R_p is only as good as measurements of imported and exported P. Furthermore, steady state values are required if R_{exp} is to be representative of a particular lake. Here no attempts have been made to quantify the effect of deviation from steady state. Valid questions are: Can R_{exp} be related statistically to lake properties such as mean depth and hydraulic washout coefficient? Do these statistically derived relationships have theoretical foundation?

Kirchner and Dillon (1975) obtained a good

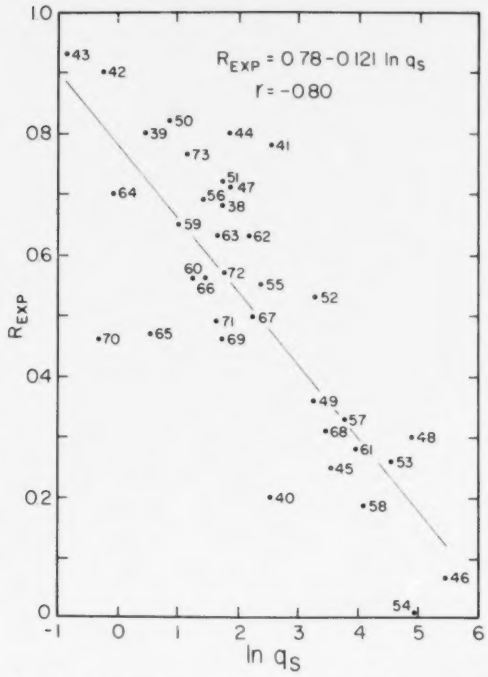

FIG. 2. Regression of R_{exp} on ln q_s for lakes summarized in Table 2.

relationship between R_{exp} and the areal water loading, q_s ($= \rho_w \cdot z$; see Table 4), but did not obtain an expected good relationship between R_{exp} and the hydraulic washout coefficient.

Chapra (1975) and Dillon and Kirchner (1975) subsequently showed that a relationship of the form

$$R_{exp} = \frac{v}{v + q_s} \tag{4}$$

where v is an apparent settling velocity (meters per year) fit the data well. This expression can be derived from the definition of $R_p (= \sigma_p \cdot \bar{z} / [\sigma_p \cdot \bar{z} + \rho_w \cdot \bar{z}] = v/[v + q_s]$, Dillon and Kirchner 1975).

Data summarized in Table 2 were analyzed to determine whether similar relationships existed, whether alternative relationships were equally valid, and particularly whether R_{exp} was related to ρ_w. Only lower mesotrophic or oligotrophic lakes were examined because increasing levels of productivity might decrease a lake's P retention capacity by providing an increasingly larger internal supply of P as anaerobic conditions develop (Vollenweider 1975) thus obscuring relationships between R_{exp} and other lake properties.

TABLE 3. Correlations between R_{exp} and various transformations and combinations of ρ_w and $\bar{z}$ for 20 lakes whose $[\bar{p}] \leq 25$ μg/liter (Table 2).

Correlation R_{exp} vs.	r	Correlation R_{exp} vs.	r
ρ_w	−0.74	$\frac{\bar{z}}{\rho_w}$	0.57
$\bar{z}$	0.33		
$\frac{1}{\rho_w}$	0.68	$\frac{\rho_w}{\bar{z}}$	0.00
$\frac{1}{\bar{z}}$	0.18		
ln ρ_w	−0.77	ln $\frac{\bar{z}}{\rho_w}$	0.58
ln $\bar{z}$	0.09	$\bar{z}^2$	0.34
$\rho_w \cdot \bar{z}$	−0.80	ρ^2_w	−0.69
ln $\rho_w \cdot \bar{z}$	−0.92	$\frac{1}{\bar{z}^2}$	0.25
$\frac{1}{\rho_w \cdot \bar{z}}$	0.75	$\frac{1}{\rho^2_w}$	0.55

Initial examination revealed a high correlation between R_{exp} and ln q_s, as summarized in Fig. 2. However, considerable variability is evident. To try to reduce this variability by including only those lakes likely to be oligotrophic and to which P loadings probably had not undergone recent changes, data from those lakes whose inflowing P concentration was ≤ 25 μg/liter were analyzed further. Table 3 summarizes the correlations and the correlation coefficients for the 20 lakes selected.

For these lakes, the highest correlation existed between R_{exp} and ln q_s (Fig. 3, $r = -0.92$). Note that slightly more than half the lakes used for these correlations were used by Kirchner and Dillon (1975). A least squares linear fit to the data provided the relationship:

$$R_{exp} = 0.86 - 0.143 \ln q_s. \tag{5}$$

The observation that R_{exp} correlated better with areal water loading than with hydraulic washout coefficient (Kirchner and Dillon 1975) is also supported by the results presented in Table 3. However, if values for Lakes Raven and Talbot are excluded as outliers, a high correlation is obtained between R_{exp} and ln ρ_w (Fig. 4, $r = -0.91$). The best fit linear regression equation was

$$R_{exp} = 0.482 - 0.112 \ln \rho_w. \tag{6}$$

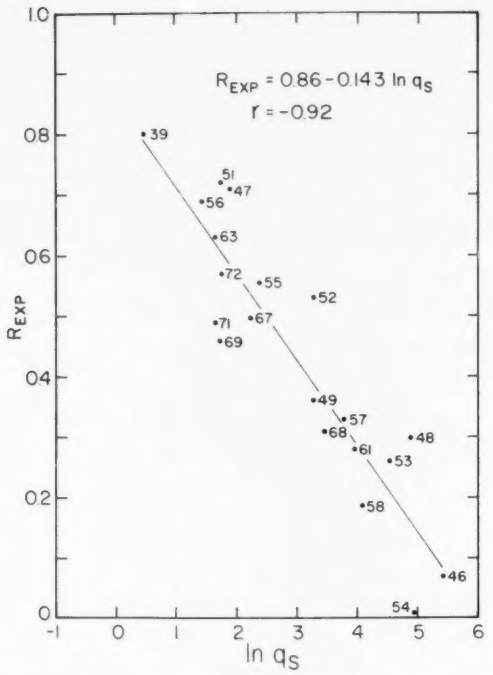

FIG. 3. Regression of R_{exp} on $\ln q_s$ for lakes whose $[\bar{p}]$ was $\leqslant 25$ μg/liter (Table 2).

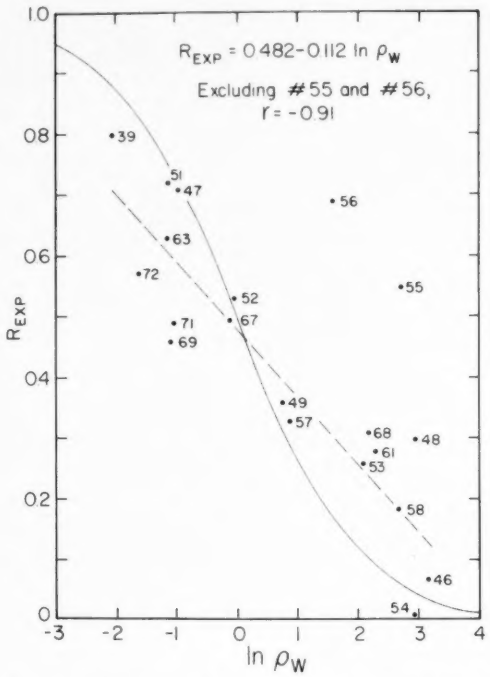

FIG. 4. Regression of R_{exp} on $\ln \rho_w$ for lakes whose $[\bar{p}]$ was $\leqslant$ 25 μg/liter (Table 2). Dashed line is regression equation excluding Raven (No. 55) and Talbot (No. 56). Solid line is a graph of $R_p = \frac{\sigma_p}{\sigma_p + \rho_w}$ for $\sigma_p = 1.0$ yr^{-1}.

Raven and Talbot are the shallowest lakes ($\bar{z} = 0.73$ and 0.85 m, respectively) in the data set so perhaps this relationship between R_{exp} and $\ln \rho_w$ is not valid for extremely shallow lakes. Dillon and Rigler (1974), testing a simple phosphorus mass balance model, reported that predicted and observed P values differed markedly for Lakes Raven and Talbot.

Although equations (5) and (6) provide a good fit to observations, they predict R_p greater than 1 for low values of q_s (<0.36 m/yr) or ρ_w (<0.01 yr^{-1}) and R_p less than 0 for high values of q_s (>395 m/yr) and ρ_w (>74 yr^{-1}). For example, equation (6) predicts $R_p = 1.22$ for Lake Tahoe. Predictions of $R_p < 0$ or > 1 are contrary to limnological experience except in unusual circumstances, thus equations (5) and (6) are not entirely adequate.

Figure 4 also includes a graph of $R_p = \frac{\sigma_p}{\sigma_p + \rho_w}$ for $\sigma_p = 1.0$ yr^{-1} to compare the data spread with the shape of a theoretical R_p vs $\ln \rho_w$ curve (at $\sigma_p = 1.0$ yr^{-1}). For $\ln \rho_w > 0$ the curve evidently underestimates observed values of R_{exp} and overestimates observed values for $\ln \rho_w < 0$.

Because σ_p and ρ_w seemed related, $\ln \sigma_p$ was regressed on $\ln \rho_w$. Direct measures of σ_p are not available for these lakes. Therefore, σ_p was determined as $\frac{\rho_w \cdot R_{exp}}{1 - R_{exp}}$ (Dillon and Rigler 1974). Data for lakes Tahoe and Superior were included because they provided data points at extremely low values of ρ_w, but lakes Raven and Talbot were excluded. The relationship

$$\ln \sigma_p = \ln 0.761 + 0.472 \ln \rho_w \qquad (r = 0.84) \quad (7)$$

or

$$\sigma_p = 0.761\ \rho_w^{0.472}$$

resulted. Substituting into $\left(R_p = \frac{\sigma_p}{\sigma_p + \rho_w}\right)$ and simplifying provided an equation of the form

$$R_p = \frac{1}{1 + \alpha \rho_w^{\beta}} \qquad (8)$$

where $\alpha = 1.3$ and $\beta = 0.4$.

TABLE 4. Summary of various relationships between R_p and ρ_w or q_s for 20 selected lakes (indicated by asterisks in Table 2). The correlation coefficients (r) in parenthesis are values obtained when lakes Superior and Tahoe are deleted.

$R_p = \frac{1}{1 + \rho_w^{1/2}}$	$r = 0.94$ (0.91)
$R_p = 0.482 - 0.112 \ln \rho_w$	$r = 0.93$ (0.91)
$R_p = \frac{11.73}{11.73 + q_s}$	$r = 0.93$ (0.89)
$R_p = 0.426e^{(-0.271\, q_s)} + 0.574e^{(-0.00949\, q_s)}$ (Kirchner and Dillon 1975)	$r = 0.94$ (0.90)
$R_p = 0.854 - 0.142 \ln q_s$	$r = 0.94$ (0.91)

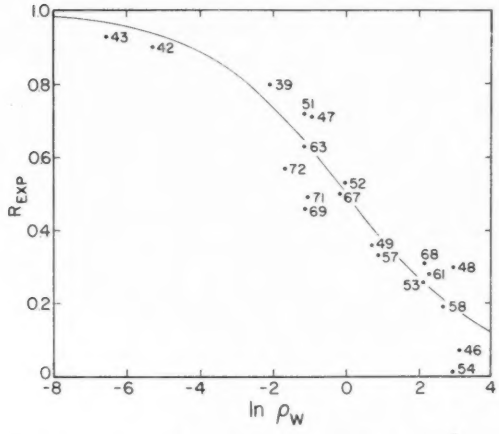

FIG. 5. Comparison of solution of $R_p = \frac{1}{1 + \rho_w^{1/2}}$ with R_{exp} for lakes whose $[\bar{p}]$ was ≤ 25 μg/liter. Raven and Talbot have been excluded; Tahoe ($[\bar{p}]$) = 100 μg/liter) and Superior ($[\bar{p}]$) = 40 μg/liter) have been added.

The coefficients α and β were also estimated using a Gauss–Newton nonlinear least squares algorithm available from the Oregon State University Computer Library. The best estimate for α was 1.12 and for β, 0.49. Thus, equation (8) can be approximated by the expression (Fig. 5):

$$R_p = \frac{1}{1 + \rho_w^{1/2}} \quad (r = 0.94). \qquad (9)$$

The best fit value for v (from equation 4) was 11.73 m/yr ($r = 0.93$), which is slightly lower than that presented by Chapra (1975) or Dillon and Kirchner (1975).

The solutions of these two expressions do not produce values of $R_p < 0$ or > 1 for extremely high and extremely low values of ρ_w or q_s as do several of the previous expressions.

Table 4 summarizes the various expressions that relate the P retention capacity to lake properties. Expressions presented are all based upon water flow normalized to lake size, as either surface area or volume, the lake attributes that seem to control the P retention coefficient to the greatest degree in these lakes.

Discussion

A graph of $[\bar{p}]$ vs. R_p is another way of expressing a relationship between trophic state and those variables thought to control trophic state. Vollenweider (1968) first proposed a graph of L_p vs. $\bar{z}$ and later modified it as L_p vs. $\bar{z}/\tau_w$ ($= q_s$; $\tau_w = \rho_w^{-1}$ = hydraulic retention time) (1975); Dillon (1975) suggested $L_p(1 - R_p)/\rho_w$ vs. $\bar{z}$, which develops directly from the steady state solution of Vollenweider's mass balance model ($[P]_x = L_p(1 - R_p)/\rho_w \cdot \bar{z}$; Dillon and Rigler 1974). The graph of $[\bar{p}]$ vs. R_p combines these terms in a different manner so that the vertical axis, $[\bar{p}]$, incorporating both P supply and water flow, represents a potential lake P concentration or trophic state. It is the concentration that would be attained if P acted as a conservative substance in lakes. This potential concentration differs from that defined by Edmondson (1961) which is the phosphorus volumnar loading, J/V. The horizontal axis expresses all those processes affecting the ability of lakes to assimilate that potential as an incorporation of P in the sediments. The relative positions of lakes on a graph of $[\bar{p}]$ vs. R_p is the same as on a $L_p(1 - R_p)/\rho_w$ vs. $\bar{z}$ graph because both graphs are based upon the same concepts and express the same information. We found it more convenient to separate those processes affecting potential lake P concentration (phosphorus supply relative to water supply) occurring external to the lake from those characteristics exhibited by a lake which control sedimentation of P, expressed by R_p.

A comparison of lakes Superior and Zurichsee

illustrates the relative importance of $[\bar{p}]$ and R_p. Lake Superior ($R_{exp} = 0.9$, $[\bar{p}] = 40$ μg/liter, mean lake P = 3–5 μg/liter) is oligotrophic (Beeton and Chandler 1966; Patalas 1972; Dobson et al. 1974), while Zurichsee ($R_{exp} = 0.25$, $[\bar{p}] = 40$ μg/liter, [P] = 32 μg/liter at vernal circulation) is eutrophic (Vollenweider 1968). Assuming that both lakes approximate a steady state, Lake Superior's ability to assimilate incoming P allows it to manifest oligotrophic characteristics while Zurichsee's much lower assimilatory ability dictates that the same $[\bar{p}]$ results in eutrophic conditions. Lake Superior probably could absorb a 40–50 μg/liter increase in $[\bar{p}]$ and remain oligotrophic, while a similar increase in $[\bar{p}]$ for Zurichsee probably would aggravate an already eutrophic problem. Slight changes in Zurichsee's assimilatory capacity would only slightly affect the steady state P concentrations, while a slight reduction in Superior's retention capacity — for example, from 0.9 to 0.8 — could double the steady state lake concentration. This sensitivity of lakes with high R_p to changes in R_p suggests that a careful determination of R_p is necessary if the results of influent P changes are to be reasonably predicted for these lakes.

The equations summarized in Table 4 show that most of the variability in the P retention capacity of these lakes can be accounted for by variation in water flow normalized to a measure of lake size, as ρ_w or q_s. The values predicted from these equations might provide an upper limit for R_p because lakes that were likely to be oligotrophic were intentionally chosen to generate the relationships. No attempt has been made to quantify the probable reduction R_p due to increased productivity. Other factors such as lake morphometry, climate, or lake chemistry could also influence R_p, and a more thorough investigation is necessary to determine these effects.

If loading and flow data are available, the expression of trophic characteristics in terms of $[\bar{p}]$ and R_p not only allows a rapid assessment of trophic state, but also provides a preliminary guide useful for management decisions. Effects of alternatives such as loading reduction or nutrient dilution can be expressed as changes in $[\bar{p}]$. Furthermore, the effect of manipulations affecting R_p, if quantifiable in terms of R_p, can be assessed. Thus, the results of nutrient inactivation or hypolimnetic aeration might be determined as increases in R_p and, therefore, as reductions in steady state P levels. In addition, the effect of combined treatments can be predicted through their changes in $[\bar{p}]$ and R_p.

The $[\bar{p}]$ vs. R_p graph is another way of expressing the relationship between trophic state and variables controlling trophic state. It develops from a continuity equation for P (equation [1]) and can be derived from Vollenweider-type mass balance models. Mass balance models of this nature are intended to describe general characteristics of phosphorus dynamics in lakes. Thus, annual averages of the input variables and rate coefficients are used and results are compared with average conditions in lakes. This type of model has been used to explain existing conditions (Vollenweider 1969, 1975) or to predict changes as a result of reducing P supplies to lakes (Sonzogni et al. 1974; Sonzogni and Lee 1974; Megard 1970). Slightly more complex mass balance models developed by Imboden (1973, 1974) and Snodgrass and O'Melia (1975) might provide a closer fit to average conditions and, more importantly to seasonal changes in lake P. However, the models require further testing to ascertain their generality.

Acknowledgments

We thank Charles O'Melia for pointing out that influent concentrations rather than loading in itself were important. It is from his comments that these inquiries developed. K. Malueg, J. Gakstatter, N. Vick, and H. Longaker provided useful comments on the manuscript.

Beeton, A. M., and D. C. Chandler. 1966. The St. Lawrence Great Lakes. p. 535–558. *In* D. G. Frey [ed.] Limnology in North America. University of Wisconsin Press, Madison, Wis.

Chapra, S. C. 1975. Comment on 'An empirical method of estimating the retention of phosphorus in lakes' by W. B. Kirchner and P. J. Dillon. Water Resour. Res. 11: 1033–1034.

Dillon, P. J. 1974. A critical review of Vollenweider's nutrient budget model and other related models. Water Resour. Bull. 10: 969–989.

1975. The phosphorus budget of Cameron Lake, Ontario: the importance of flushing rate to the degree of eutrophy of lakes. Limnol. Oceanogr. 20: 28–39.

Dillon, P. J., and W. B. Kirchner. 1975. Reply. Water Resour. Res. 11: 1035–1036.

Dillon, P. J., and F. H. Rigler. 1974. A test of a simple nutrient budget model predicting the phosphorus concentration in lake water. J. Fish. Res. Board Can. 31: 1771–1778.

Dobson, H. F. H., M. Gilbertson, and P. G. Sly. 1974. A summary and comparison of nutrients and related water quality in Lakes Erie, Ontario, Huron, Superior. J. Fish. Res. Board Can. 31: 731–738.

Edmondson, W. T. 1961. Changes in Lake Washington following an increase in the nutrient income. Verh. Int. Ver. Limnol. 14: 167–175.

Imboden, D. M. 1973. Limnologische Transport- und Nährstoffmodelle. Schweiz. Z. Hydrol. 35: 29–68.

1974. Phosphorus model of lake eutrophication. Limnol. Oceanogr. 19: 297–305.

KIRCHNER, W. B., AND P. J. DILLON. 1975. An empirical method of estimating the retention of phosphorus in lakes. Water Resour. Res. 11: 182–183.

LERMAN, A. 1974. Eutrophication and water quality of lakes: control by water residence time and transport to sediments. Hydrol. Sci. Bull. 19: 25–34.

LORENZEN, M. W. 1973. Predicting of effects of nutrient diversion in lake recovery. p. 205–210. *In* E. J. Middlebrooks, D. H. Falkenborg, and T. E. Maloney [ed.] Modeling the eutrophication process. Ann Arbor Science, Ann Arbor, Mich.

MEGARD, R. O. 1970. Lake Minnetonka: nutrients, nutrient abatement, and the photosynthetic system of the phytoplankton. Limnol. Res. Cent., Univ. Minnesota, Minneapolis, Minn. Interim Rep. 7: 214 p.

PATALAS, K. 1972. Crustacean plankton and the eutrophication of the St. Lawrence Great Lakes. J. Fish. Res. Board Can. 29: 1451–1462.

RIGLER, F. H. 1975. Nutrient kinetics and the new typology. Verh. Int. Ver. Limnol. 19: 197–210.

SAWYER, C. N. 1947. Fertilization of lakes by agricultural and urban drainage. J. N. Engl. Water Works Assoc. 61: 109–127.

SNODGRASS, W. J., AND C. R. O'MELIA. 1975. Predictive model for phosphorus in lakes. Environ. Sci. Technol. 9: 937–944.

SONZOGNI, W. C., AND G. F. LEE. 1974. Diversion of wastewater from Madison Lakes. J. Environ. Eng. Div., Am. Soc. Civ. Eng. 100: 153–170.

SONZOGNI, W. C., P. D. UTTORMARK, AND G. F. LEE. 1976. The phosphorus residence time model: theory and application. Water Res. 10: 429–435.

TREUNERT, E., A. WILHELMS, AND H. BERNHARDT. 1974. Einfluß der Probenentnahme-Häufigkeit auf die Ermittlung der Jahres-Phosphorfrachtwerte mittlerer Bäche. Hydrochem. Hydrogeol. Mitt. 1: 175–198.

VOLLENWEIDER, R. A. 1968. Scientific fundamentals of the eutrophication of lakes and flowing waters, with particular reference to nitrogen and phosphorus as factors in eutrophication. Organ. Econ. Coop. Dev., Paris. Tech. Rep. DAS/CSI/68.27: 159 p.

1969. Möglichkeiten und Grenzen elementarer Modelle der Stoffbilanz von Seen. Arch. Hydrobiol. 66: 1–36.

1975. Input–output models with special reference to the phosphorus loading concept in limnology. Schweiz. Z. Hydrol. 37: 53–83.